Trigonometry

Precalculus
Chapter 4

- This Slideshow was developed to accompany the textbook
  - *Precalculus*
  - *By Richard Wright*
  - *https://www.andrews.edu/~rwright/Precalculus-RLW/Text/TOC.html*
- Some examples and diagrams are taken from the textbook.

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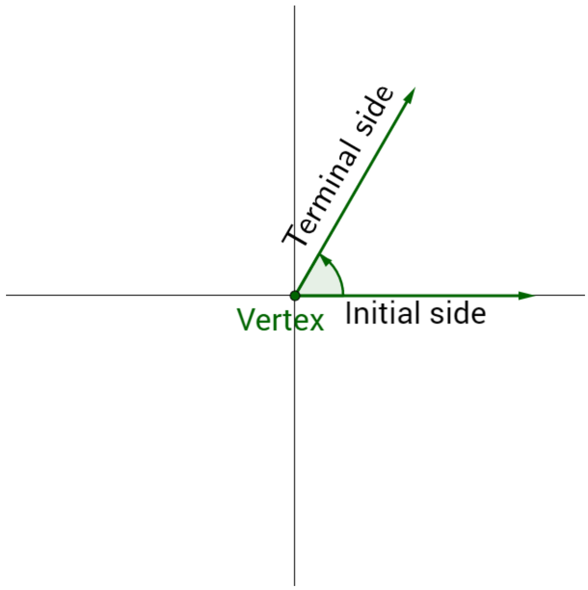
4-01 Angle Measures

In this section, you will:

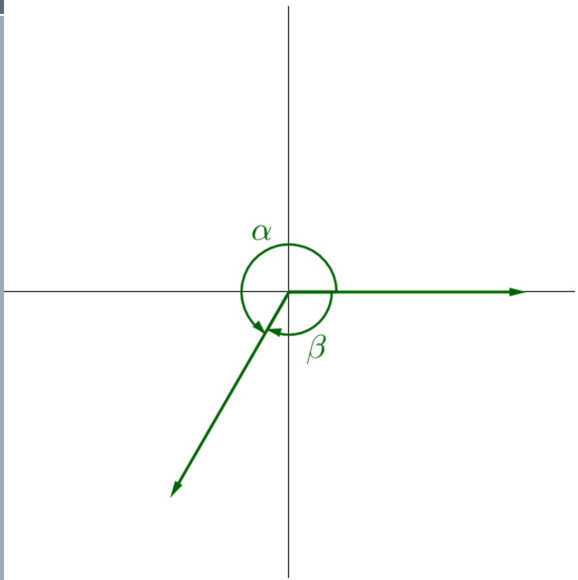
- Draw angles in standard position.
- Convert between degrees and radians.
- Find coterminal angles.
- Use applications of angles.

π 4-01 Angle Measures

- Angles in standard position
  - Vertex at origin
  - Initial side on positive x-axis
  - Terminal side rotates counterclockwise



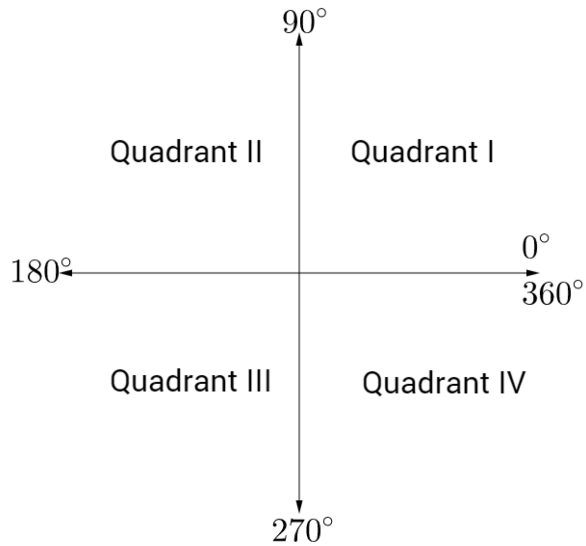
4-01 Angle Measures



- Coterminal Angles
- 2 angles with same sides, but different measures
- To find coterminal angles
  - $\theta \pm 360°$

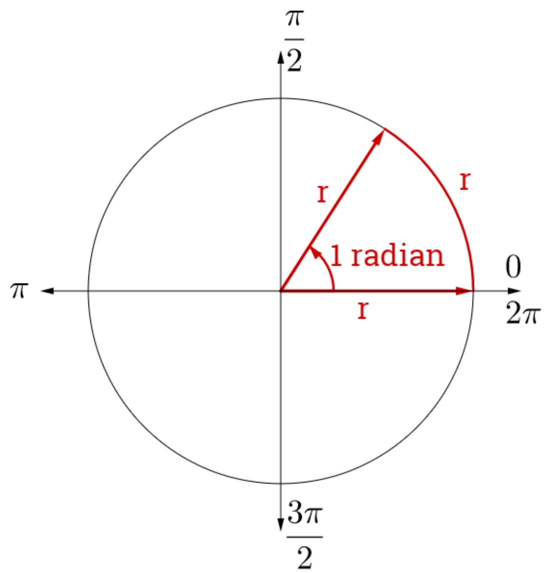
4-01 Angle Measures

› Degree Measures



4-01 Angle Measures

- Radian Measures
  - Angle where radius = arc length
- Acute → $\theta < 90°, \frac{\pi}{2}$
- Obtuse → $90° < \theta < 180°$
  - $\frac{\pi}{2} < \theta < \pi$
- Complementary → $\alpha + \beta = 90°, \frac{\pi}{2}$
- Supplementary → $\alpha + \beta = 180°, \pi$



4-01 Angle Measures

- Find a coterminal angle with $\theta = -\frac{\pi}{8}$
- Find the supplement of $\theta = \frac{\pi}{4}$

Coterminal

$$-\frac{\pi}{8} \pm 2\pi = -\frac{\pi}{8} \pm \frac{16\pi}{8} = -\frac{17\pi}{8}, \frac{15\pi}{8}$$

Supplement

$$S + \frac{\pi}{4} = \pi$$

$$S = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

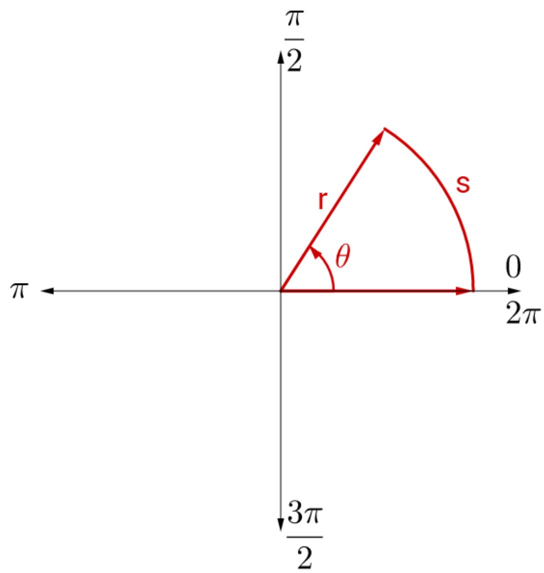
4-01 Angle Measures

- Convert radians to degrees
  - 180° = π
- Convert 120° to radians

$$120°\left(\frac{\pi}{180°}\right) = \frac{120\pi}{180} = \frac{2\pi}{3}$$

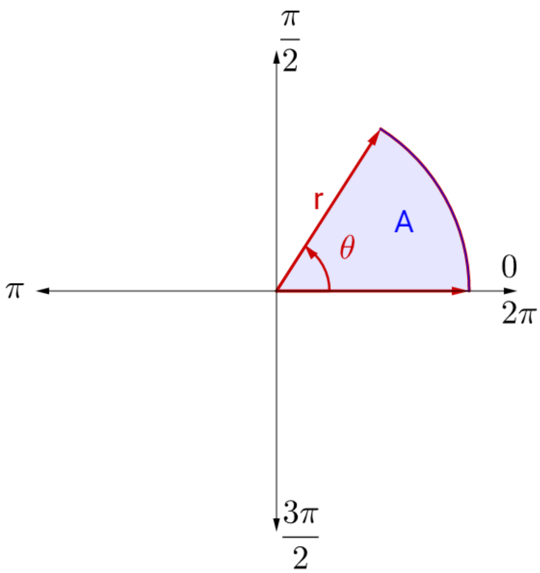
4.01 Angle Measures

- Applications
- Arc Length
  - $S = r\theta$
    - Where θ is in radians



4-01 Angle Measures

- Area of Sector
- A = fraction of circle × πr^2
- $A = \frac{\theta}{2\pi} \times \pi r^2$
- $A = \frac{1}{2}\theta r^2$
  - Where θ is in radians



4-01 Angle Measures

- Speeds
  - Angular speed: $\omega = \frac{\theta}{t}$
  - Linear speed (tangential): $v = \frac{s}{t}$
    - $v = \frac{r\theta}{t}$
    - $v = r\omega$

4-01 Angle Measures

- A 6-inch diameter gear makes 2.5 revolutions per second. Find the angular speed in radians per second.

- How fast is a tooth at the edge of the gear moving in in./s?

$$\frac{2.5\ rev}{s}\left(\frac{2\pi\ rad}{1\ rev}\right) = 5\pi\frac{rad}{s}$$

$$v = rw$$

$$v = (3\ in.)\left(5\pi\frac{rad}{s}\right) = 15\pi\ in./s$$

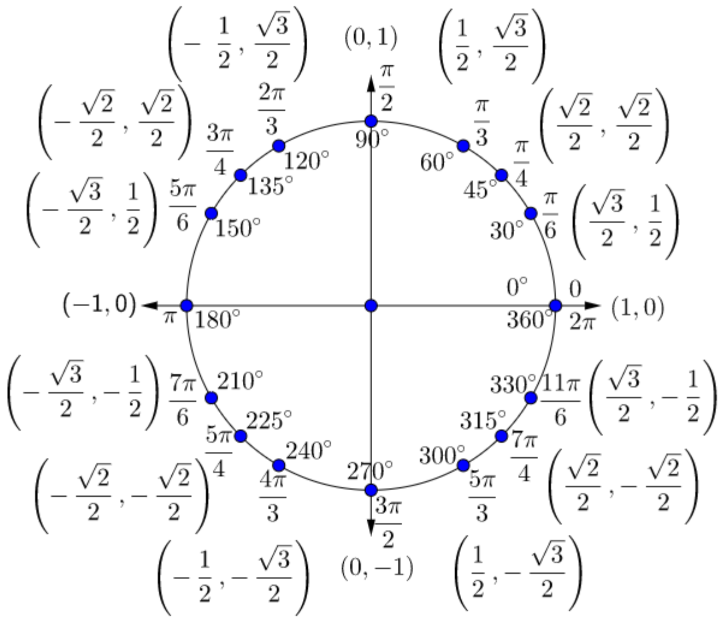
4-02 Unit Circle

In this section, you will:

- Understand the unit circle.
- Use the unit circle to evaluate trigonometric functions.
- Use even and odd trigonometric functions.
- Use a calculator to evaluate trigonometric functions.

4-02 Unit Circle

- Unit circle
  - $r = 1$
  - $x^2 + y^2 = 1$



4-02 Unit Circle

- Trigonometric Functions (Unit circle)
- $\sin t = y$
  - sine
- $\cos t = x$
  - cosine
- $\tan t = \frac{y}{x}$
  - tangent
- $\csc t = \frac{1}{y}$
  - cosecant
- $\sec t = \frac{1}{x}$
  - secant
- $\cot t = \frac{x}{y}$
  - cotangent

4-02 Unit Circle

› Evaluate 6 trig functions of $t = \frac{2\pi}{3}$

$$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{2\pi}{3} = -\frac{1}{2}$$

$$\tan\frac{2\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$\csc\frac{2\pi}{3} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec\frac{2\pi}{3} = \frac{1}{-\frac{1}{2}} = -2$$

$$\cot\frac{2\pi}{3} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

4-02 Unit Circle

- Evaluate
- $\sec\frac{4\pi}{3}$
- $\sin 2\pi$
- $\tan\frac{\pi}{2}$
- $\csc\frac{11\pi}{6}$
- $\cot\frac{3\pi}{4}$
- $\cos 0$

Draw angles on unit circle for reference

$$\sec\frac{4\pi}{3}=\frac{1}{x}=\frac{1}{-\frac{1}{2}}=-2$$

$$\sin 2\pi=y=0$$

$$\tan\frac{\pi}{2}=\frac{y}{x}=\frac{1}{0}=undefined$$

$$\csc\frac{11\pi}{6}=\frac{1}{y}=\frac{1}{-\frac{1}{2}}=-2$$

$$\cot\frac{3\pi}{4}=\frac{x}{y}=\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=-1$$

$$\cos 0=x=1$$

4-02 Unit Circle

- Evaluate
- $\sin\left(-\frac{2\pi}{3}\right)$
- $\sin\left(-\frac{11\pi}{2}\right)$
- $\cos\frac{9\pi}{3}$

Find coterminal angles between 0 and 2π

$$\sin\left(-\frac{2\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = y = -\frac{\sqrt{3}}{2}$$

$$\cos\frac{9\pi}{3} = \cos\pi = x = -1$$

$$\sin\left(-\frac{11\pi}{2}\right) = \sin\frac{\pi}{2} = y = 1$$

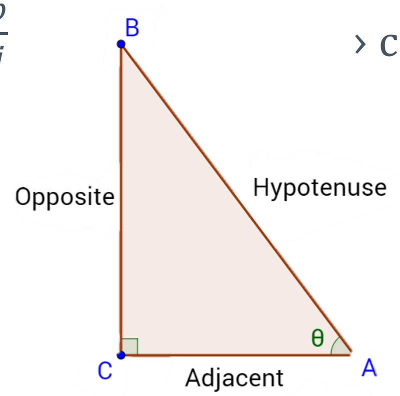
4-03 Right Triangle Trigonometry

In this section, you will:

- Use right triangles to evaluate trigonometric functions.
- Use special right triangles to evaluate trigonometric functions of common angles.

4-03 Right Triangle Trigonometry

- $\sin A = \frac{opp}{hyp}$
- $\cos A = \frac{adj}{hyp}$
- $\tan A = \frac{opp}{adj}$
- $\csc A = \frac{hyp}{opp}$
- $\sec A = \frac{hyp}{adj}$
- $\cot A = \frac{adj}{opp}$

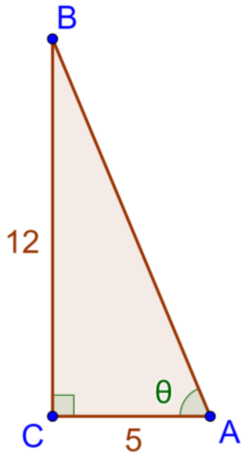


SOH
CAH
TOA

SOH
CAH
TOA

4-03 Right Triangle Trigonometry

- Find the values of the six trig functions



$$hyp = \sqrt{5^2 + 12^2} = 13$$

$$\sin\theta = \frac{12}{13}$$

$$\cos\theta = \frac{5}{13}$$

$$\tan\theta = \frac{12}{5}$$

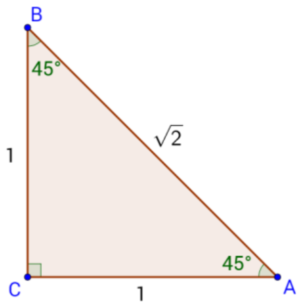
$$\csc\theta = \frac{13}{12}$$

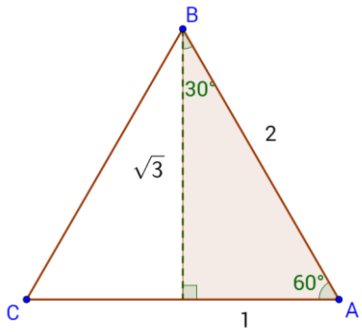
$$\sec\theta = \frac{13}{5}$$

$$\cot\theta = \frac{5}{12}$$

4-03 Right Triangle Trigonometry

- Special right triangles





- $\sin\frac{\pi}{4}$
- $\csc\frac{\pi}{3}$
- $\tan 30°$

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\csc\frac{\pi}{3} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

4-03 Right Triangle Trigonometry

› Sketch a triangle and find the other 5 trig functions

› $\tan\theta = 3$

$$\tan\theta = 3 = \frac{3}{1} = \frac{opp}{adj}$$

$$\sin\theta = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\tan\theta = 3$$

$$\csc\theta = \frac{\sqrt{10}}{3}$$

$$\sec\theta = \frac{\sqrt{10}}{1} = \sqrt{10}$$

$$\cot\theta = \frac{1}{3}$$

4-04 Right Triangle Trigonometry and Identities

In this section, you will:

- Use right triangles to evaluate trigonometric functions.
- Use special right triangles to evaluate trigonometric functions of common angles.

4-04 Right Triangle Trigonometry and Identities

- Basic Identities
- Reciprocal
- $\sin u = \frac{1}{\csc u}$ $\cos u = \frac{1}{\sec u}$ $\tan u = \frac{1}{\cot u}$
- $\csc u = \frac{1}{\sin u}$ $\sec u = \frac{1}{\cos u}$ $\cot u = \frac{1}{\tan u}$
- Quotient
- $\tan u = \frac{\sin u}{\cos u}$ $\cot u = \frac{\cos u}{\sin u}$
- Pythagorean
- $\sin^2 u + \cos^2 u = 1$ $1 + \tan^2 u = \sec^2 u$ $\cot^2 u + 1 = \csc^2 u$
- Note: $\sin^2 u = (\sin u)^2$

4-04 Right Triangle Trigonometry and Identities

- Cofunction Identities

- $\sin(90° - u) = \cos u$
- $\tan(90° - u) = \cot u$
- $\sec(90° - u) = \csc u$
- $\cos(90° - u) = \sin u$
- $\cot(90° - u) = \tan u$
- $\csc(90° - u) = \sec u$

4-04 Right Triangle Trigonometry and Identities

- Let θ be an acute angle such that $\cos\theta = 0.96$
- Find $\sin\theta$
- Find $\tan\theta$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + 0.96^2 = 1$$
$$\sin^2\theta + 0.9216 = 1$$
$$\sin^2\theta = 0.0784$$
$$\sin\theta = 0.28$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$= \frac{0.28}{0.96}$$
$$= 0.2917$$

These could also have been solved using right triangles

4-04 Right Triangle Trigonometry and Identities

- Let β be an acute angle such that $\tan\beta = 4$
- Find $\cot\beta$
- $\sec\beta$

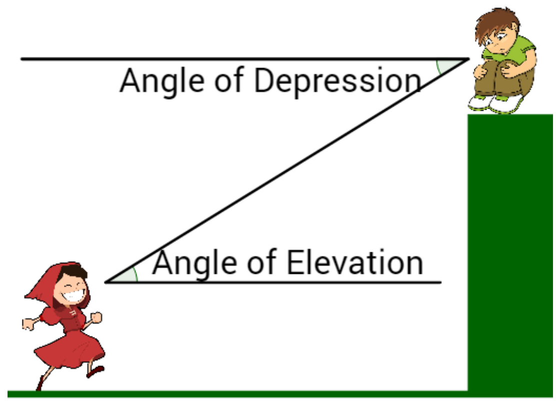
$$\cot\beta = \frac{1}{\tan\beta}$$
$$\cot\beta = \frac{1}{4}$$

$$1 + \tan^2\beta = \sec^2\beta$$
$$1 + 4^2 = \sec^2\beta$$
$$\sqrt{17} = \sec\beta$$

4-04 Right Triangle Trigonometry and Identities

- Angles of Elevation and Depression

- Both are measured from the horizontal



4-04 Right Triangle Trigonometry and Identities

- A 12-meter flagpole casts a 6-meter shadow. Find the angle of elevation of the sun.

$$\tan\theta = \frac{12}{6}$$
$$\tan\theta = 2$$
$$\theta \approx 63.4°$$

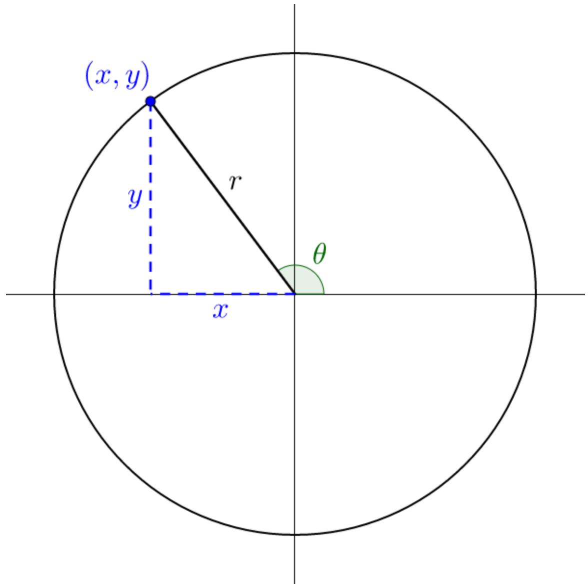
4-05 Trigonometric Functions of Any Angle

In this section, you will:
- Evaluate trigonometric functions of any angle.
- Find reference angles.

4-05 Trigonometric Functions of Any Angle

- $\sin\theta = \frac{y}{r}$ $\quad\quad$ $\csc\theta = \frac{r}{y}$
- $\cos\theta = \frac{x}{r}$ $\quad\quad$ $\sec\theta = \frac{r}{x}$
- $\tan\theta = \frac{y}{x}$ $\quad\quad$ $\cot\theta = \frac{x}{y}$

- $r = \sqrt{x^2 + y^2}$



4-05 Trigonometric Functions of Any Angle

- Let (-2, 3) be a point on the terminal side of θ. Find sine, cosine, and tangent of θ.

Use Pythagorean Theorem to find r

$$r = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$$

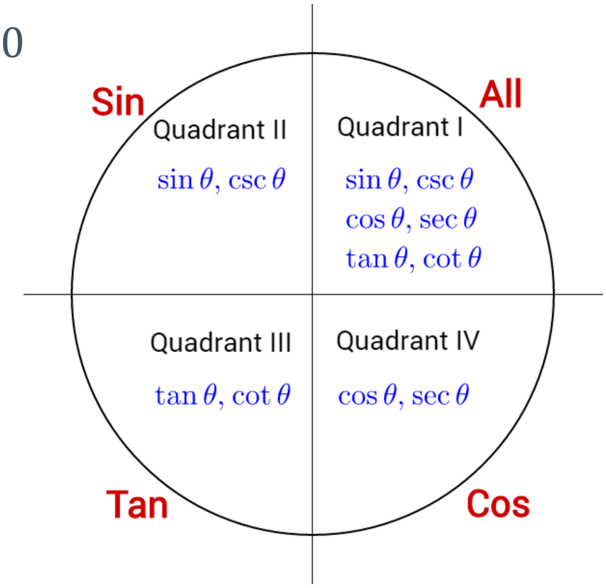
$$\sin\theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\cos\theta = -\frac{2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\tan\theta = -\frac{3}{2}$$

4-05 Trigonometric Functions of Any Angle

- Given $\sin\theta = \frac{4}{5}$ and $\tan\theta < 0$ find $\cos\theta$ and $\csc\theta$.



Quadrant II (sine +, tangent -)

$$\sin\theta = \frac{4}{5} = \frac{y}{r}$$

Use Pythagorean theorem to find r = -3

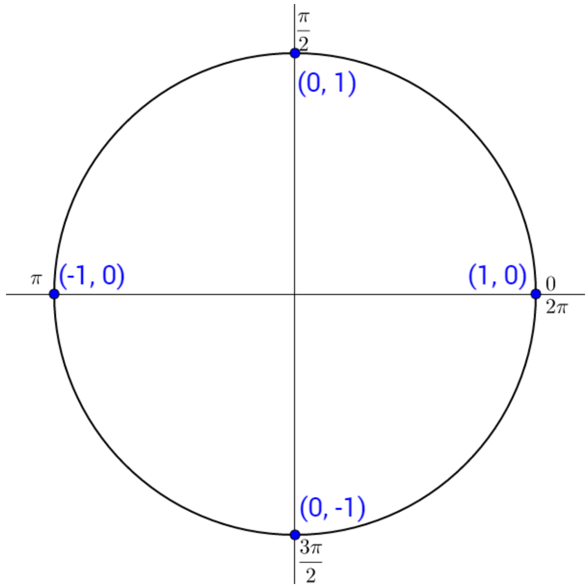
$$\cos\theta = -\frac{3}{5}$$

$$\csc\theta = \frac{5}{4}$$

4-05 Trigonometric Functions of Any Angle

› Evaluate $\sin \pi$

› $\tan \frac{\pi}{2}$



$$\sin \pi = \frac{y}{r} = \frac{0}{1} = 0$$

$$\tan \frac{\pi}{2} = \frac{y}{x} = \frac{1}{0} = undefined$$

4-05 Trigonometric Functions of Any Angle

- Reference Angle
  - Angle between terminal side and nearest x-axis
- Find the reference angle for $\frac{5\pi}{4}$
- Find the reference angle for $\frac{5\pi}{3}$

4-05 Trigonometric Functions of Any Angle

- Use a reference angle to evaluate $\cos\frac{5\pi}{3}$
- $\sin 150°$

Reference angle is $\frac{\pi}{3}$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

Quadrant IV where cos is +

Reference angle is 30°

$$\sin 30° = \frac{1}{2}$$

Quadrant II where sin is +

4-05 Trigonometric Functions of Any Angle

› Use a reference angle to evaluate $\tan \frac{11\pi}{6}$

Reference angle is $\frac{\pi}{6}$

$$\tan \frac{\pi}{6} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Quadrant IV where tan is -

4-05 Trigonometric Functions of Any Angle

- Let θ be an angle in quadrant III such that $\sin\theta = -\frac{5}{13}$. Find
- $\sec\theta$
- $\tan\theta$

$$\sin\theta = -\frac{5}{13} = \frac{y}{r}$$

$$y = -5, r = 13$$

Use Pythagorean theorem to find $x = -12$

$$\sec\theta = \frac{r}{x} = -\frac{13}{12}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{-12} = \frac{5}{12}$$

4-06 Graphs of Sine and Cosine

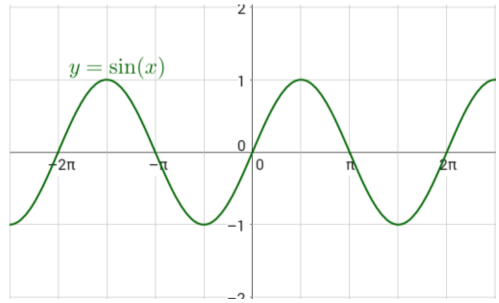
In this section, you will:

- Graph $y = \sin x$ and $y = \cos x$.
- Graph transformations of sine and cosine graphs.
- Write mathematical models using sine and cosine.

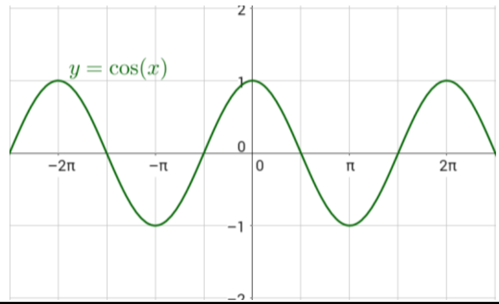
π

4-06 Graphs of Sine and Cosine

- $y = \sin x$
  - Starts at 0
  - Amplitude = 1
  - Period = 2π



- $y = \cos x$
  - Starts at 1
  - Amplitude = 1
  - Period = 2π



Point out
- Amplitude
- period
- key points

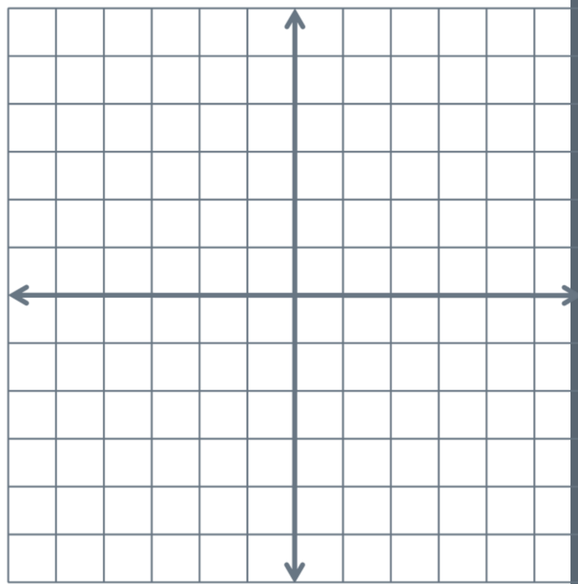
4-06 Graphs of Sine and Cosine

- Transformations
- $y = a\sin(bx - c) + d$
  - a = amplitude = vertical stretch
  - b = horizontal shrink
    - Period $T = \frac{2\pi}{b}$
  - c = horizontal shift
    - Phase shift $PS = \frac{c}{b}$ (Right if c is positive)
  - d = vertical shift
    - Midline $y = d$

c is like h
d is like k

4-06 Graphs of Sine and Cosine

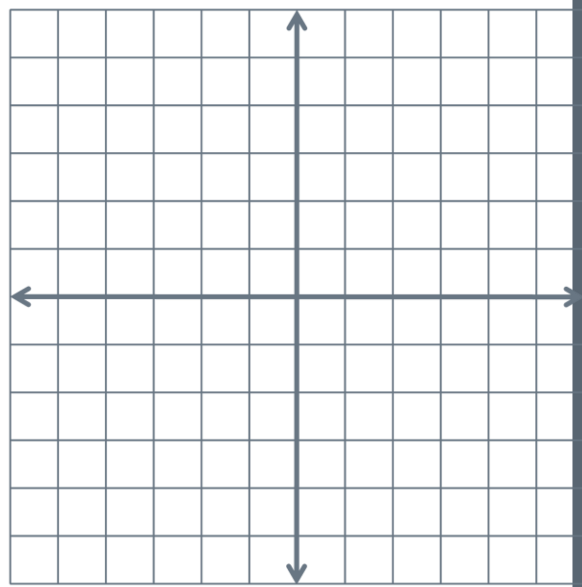
- Graph $f(x) = 2\sin x$



Same as sine, but amp = 2

4-06 Graphs of Sine and Cosine

› Graph $y = \cos\frac{x}{2}$

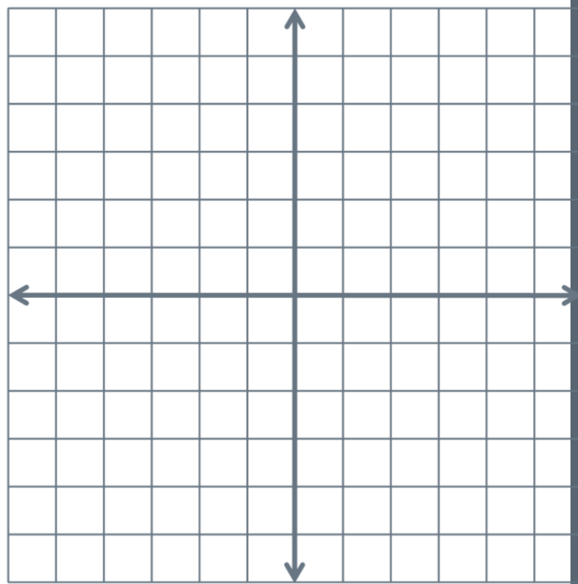


Period $T = \frac{2\pi}{b}$

$$T = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

4-06 Graphs of Sine and Cosine

- Graph $y = 2\sin\left(x - \frac{\pi}{2}\right)$



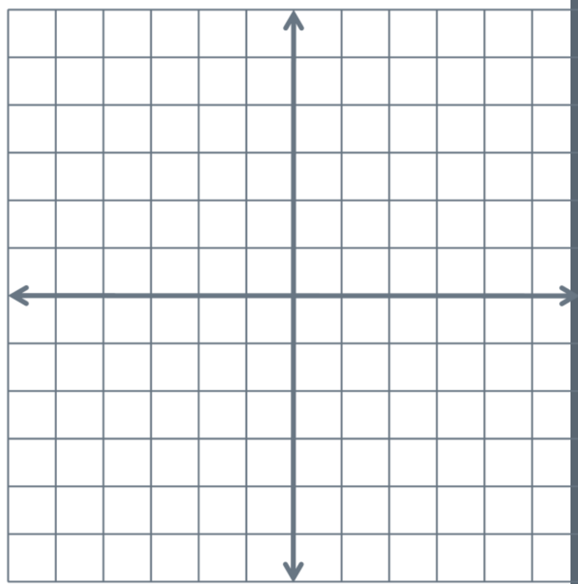
$$a = 2$$
$$b = 1 \rightarrow T = 2\pi$$
$$h = \frac{\pi}{2} \rightarrow PS = \frac{h}{b} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2} \text{ to right}$$

Draw $2\sin x$ first and then do the phase shift

4-06 Graphs of Sine and Cosine

› Graph

$$y = -\frac{1}{2}\sin(\pi x + \pi) + 1$$



$$a = -\frac{1}{2} = amp$$

$$b = \pi \rightarrow T = \frac{2\pi}{b} \rightarrow \frac{2\pi}{\pi} = 2$$

$$h = -\pi \rightarrow PS = \frac{h}{b} \rightarrow -\frac{\pi}{\pi} = -1$$

PS left 1

$$k = 1$$

Shift up 1

Graph $\frac{1}{2}\sin \pi x$ first labeling the key points with a period of 2

Reflect over the x-axis because a is negative

Shift left 1 and up 1

4-07 Graphs of Other Trigonometric Functions

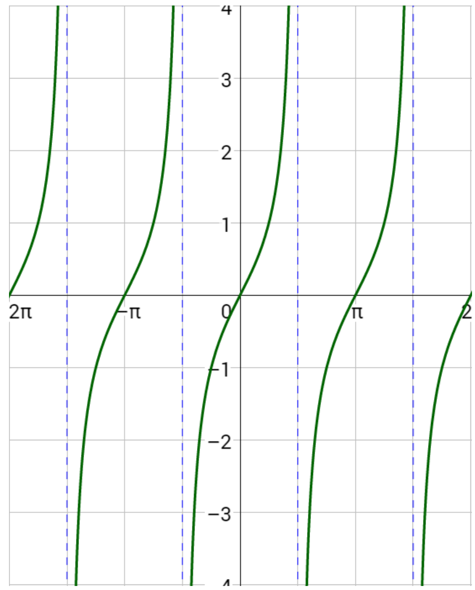
In this section, you will:

- Graph tangent, secant, cosecant, and cotangent
- Graph a damped trigonometric function

π

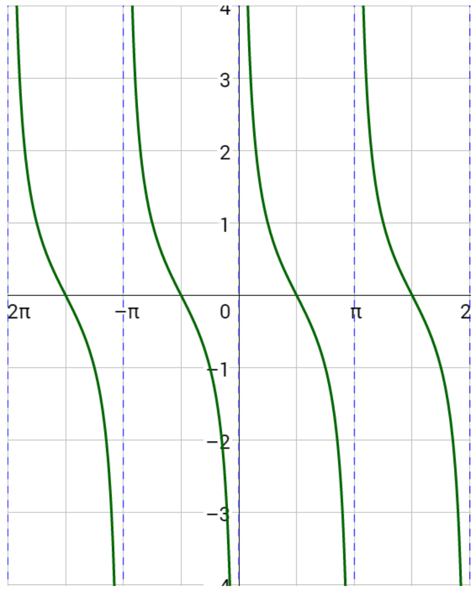
4-07 Graphs of Other Trigonometric Functions

- $y = \tan x$
  - Period = π
    - $T = \frac{\pi}{b}$
  - Asymptotes where tangent undefined, $\frac{\pi}{2}, \frac{3\pi}{2}$



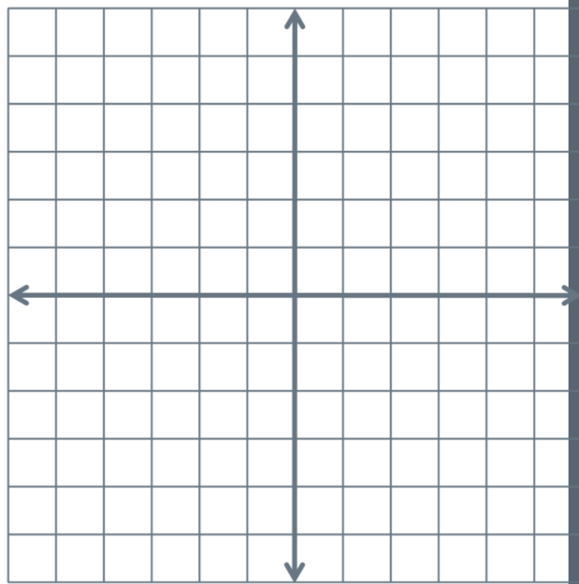
4-07 Graphs of Other Trigonometric Functions

- $y = \cot x$
  - Period = π
    - $T = \frac{\pi}{b}$
  - Asymptotes at 0, π, 2π



4-07 Graphs of Other Trigonometric Functions

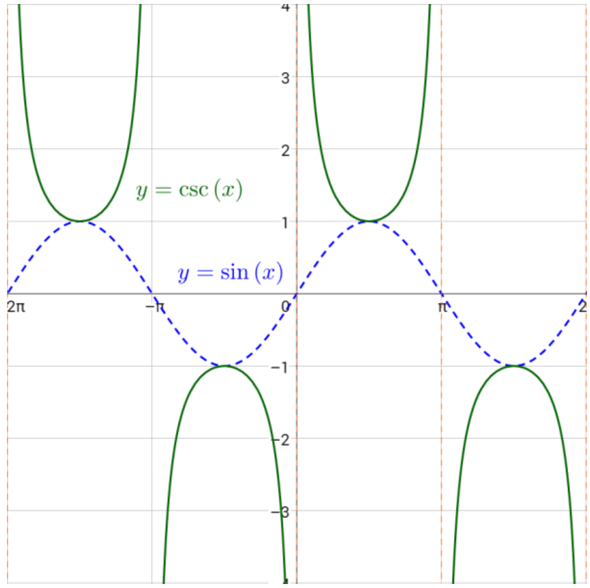
- Graph $y = \tan\frac{x}{4}$



$$b = \frac{1}{4}$$
$$T = \frac{\pi}{b} = \frac{\pi}{\frac{1}{4}} = 4\pi$$
$$a = 1$$

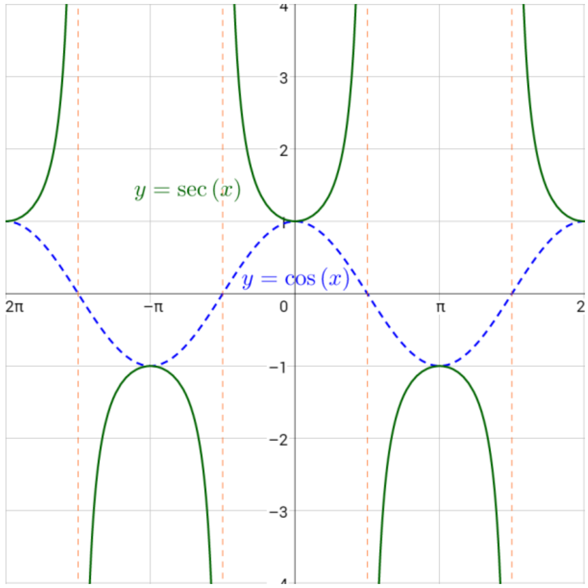
4-07 Graphs of Other Trigonometric Functions

- $y = \csc x$
  - Period = 2π
  - Asymptotes where sine = 0
    - $0, \pi, 2\pi$



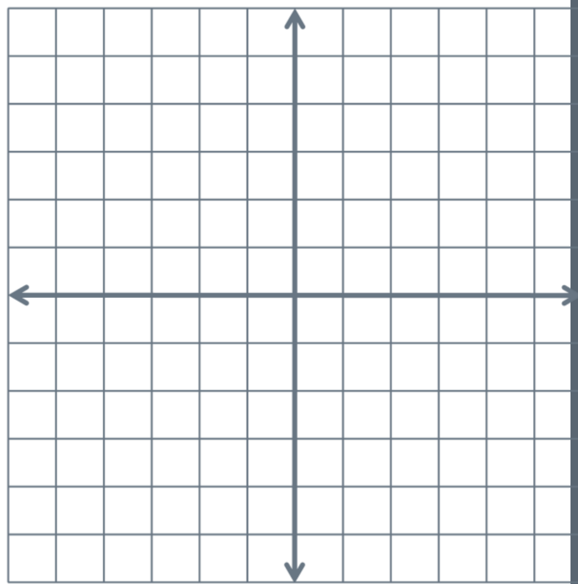
4-07 Graphs of Other Trigonometric Functions

- $y = \sec x$
  - Period = 2π
  - Asymptotes where cosine = 0
    - $\frac{\pi}{2}, \frac{3\pi}{2}$



4-07 Graphs of Other Trigonometric Functions

› Graph $y = 2\csc\left(x + \frac{\pi}{2}\right)$

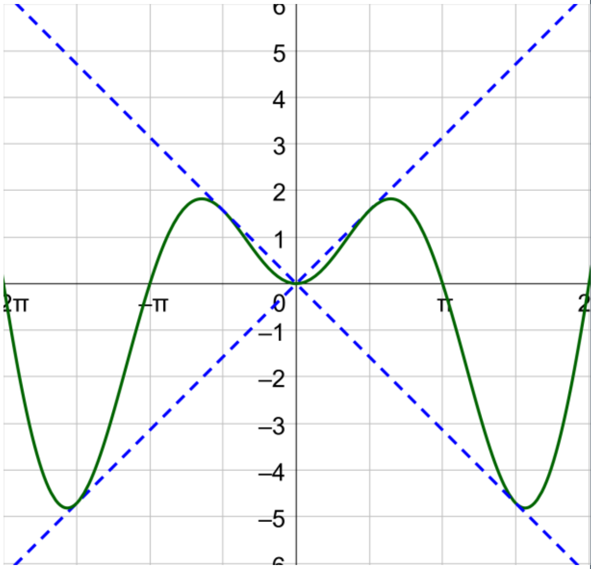


$$a = 2$$
$$b = 1$$
$$T = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$$
$$c = -\frac{\pi}{2}$$
$$PS = \frac{c}{b} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$$
$$k = 0$$

Start by graphing $2\sin x$
Then shift left $\frac{\pi}{2}$
Then draw asymptotes at the x-intercepts
Then draw csc graph

4-07 Graphs of Other Trigonometric Functions

- Damped Trig Functions
- $y = \boxed{x} \sin x$
  - The x is the damping function
  - Graph the damping function and its reflection over x-axis
  - Graph the trig between



4-08 Inverse Trigonometric Functions

In this section, you will:

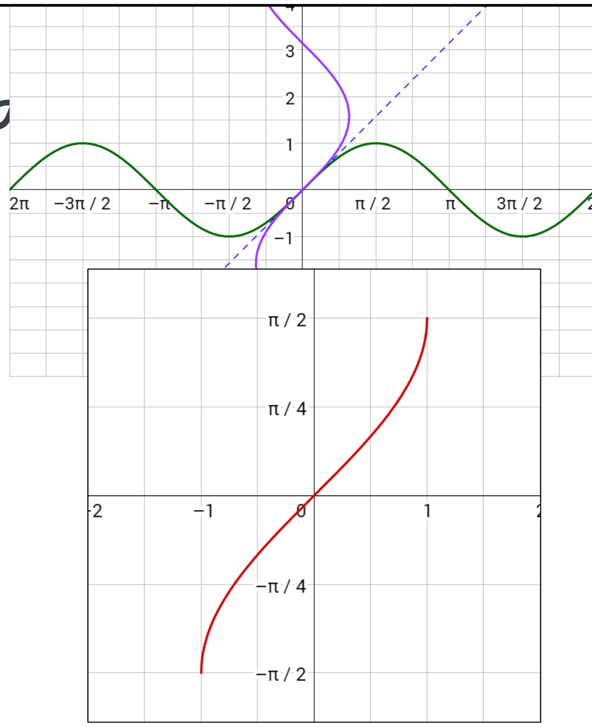
- Use the inverse sine, cosine, and tangent functions
- Evaluate inverse trigonometric functions

4-08 Inverse Trigonometric Functions

- Inverses switch *x* and *y*
  - *Reflects graph over y = x*
- $y = \sin x \leftrightarrow x = \sin^{-1} y$
- Inverse trig functions give the angle

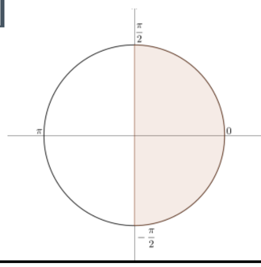
4-08 Inverse Trigonometric

- Inverse Sine
  - $y = \sin^{-1} x$
  - $y = \arcsin x$



- Domain: [-1, 1]
- Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

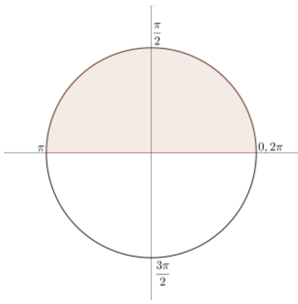
- $\arcsin(-1)$

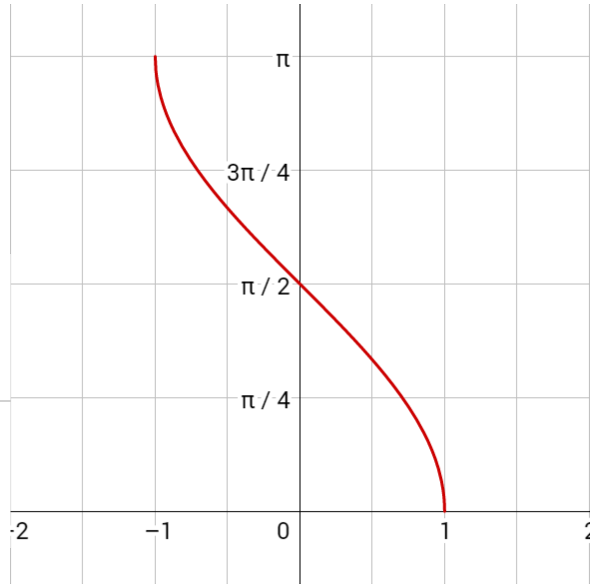


$$\arcsin(-1) = -\frac{\pi}{2}$$
$$\sin\theta = y = -1$$

4-08 Inverse Trigonometric Functions

- Inverse Cosine
  - $y = \cos^{-1} x$
  - $y = \arccos x$
- Domain: [-1, 1]
- Range: [0, π]
- $\arccos \frac{1}{2}$



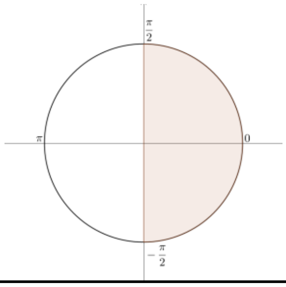


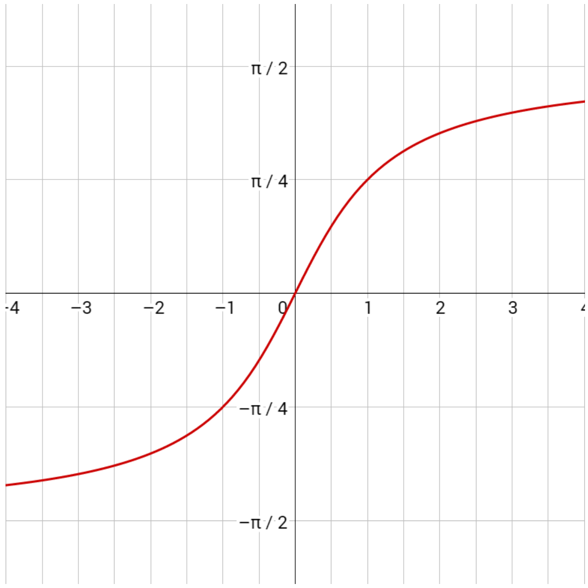
Think $\cos\theta = \frac{1}{2}$

$$\arccos \frac{1}{2} = \frac{\pi}{3}$$

4-08 Inverse Trigonometric Functions

- Inverse Tangent
  - $y = \tan^{-1} x$
  - $y = \arctan x$
- Domain: (-∞, ∞)
- Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$





4-08 Inverse Trigonometric Functions

- Evaluate
- $\sin^{-1}\left(\frac{1}{2}\right)$
- $\arcsin\sqrt{3}$

Think $\sin\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6}$

Think $\sin\theta = \sqrt{3}$ → Not possible

4-08 Inverse Trigonometric Functions

- Evaluate
- $\cos^{-1}\frac{\sqrt{3}}{2}$
- $\arctan\frac{\sqrt{3}}{3}$

Think $\cos\theta = \frac{\sqrt{3}}{2} \rightarrow \theta = \frac{\pi}{6}$

Think $\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \rightarrow \theta = \frac{\pi}{6}$

4-09 Compositions involving Inverse Trigonometric Functions

In this section, you will:
- Evaluate compositions of inverse functions

4-09 Compositions involving Inverse Trigonometric Functions

- If $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, then $\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$

- $\tan(\arctan(-14))$

$$\sin(\arcsin \pi)$$

Check domain of inner: arctan domain $x \neq \frac{\pi}{2}n$ so -14 is in domain.
Check range of outer: tan range (-∞, ∞) so -14 is in range
Ans: -14

Check domain of inner: arcsin domain [-1, 1]
π is not in domain, so not possible

4-09 Compositions involving Inverse Trigonometric Functions

› $\arcsin\left(\sin\frac{5\pi}{3}\right)$

› $\arccos\left(\cos\frac{7\pi}{6}\right)$

Check domain of inner: sin domain (-∞, ∞) so $\frac{5\pi}{3}$ is included
Check range of outer: arcsin range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so use coterminal angle
Ans $-\frac{\pi}{3}$

Check domain of inner: cos domain (-∞, ∞) so $\frac{7\pi}{6}$ is included
Check range of outer: arccos range $[0, \pi]$ so use reference angle to find another angle with same sign and reference angle
Ans $\frac{5\pi}{6}$

4-09 Compositions involving Inverse Trigonometric Functions

› $\tan^{-1}(\cos\pi)$

› $\cos^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right)$

Check domain of inner: cos domain (-∞, ∞) so π is in domain.
Use the unit circle to find inner: $\cos\pi = -1$
Evaluate outer $\tan^{-1}(-1) = -\frac{\pi}{4}$

Check domain of inner: sin domain (-∞, ∞) so $\frac{\pi}{6}$ is in domain.
Use the unit circle to find inner: $\sin\frac{\pi}{6} = \frac{1}{2}$
Evaluate outer $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

4-09 Compositions involving Inverse Trigonometric Functions

- $\cos\left(\tan^{-1}\left(-\frac{3}{4}\right)\right)$
- $\sin\left(\cos^{-1}\left(\frac{2}{3}\right)\right)$

The input is in arctan so they are ratio of sides. Use those to make a triangle.
Use Pythagorean theorem to find r
Evaluate cos of that angle in the triangle
Ans: $\frac{4}{5}$

The input is in arccos so they are ratio of sides. Use those to make a triangle.
Use Pythagorean theorem to find y
Evaluate sin of that angle in the triangle
Ans: $\frac{\sqrt{5}}{3}$

4-09 Compositions involving Inverse Trigonometric Functions

- $\sec(\arctan x)$

The input is in arctan so they are ratio of sides. Use those to make a triangle.
Use Pythagorean theorem to find $r = \sqrt{x^2+1}$
Evaluate sec of that angle in the triangle
Ans: $\frac{\sqrt{x^2+1}}{1}$

4-10 Applications of Right Triangle Trigonometry

In this section, you will:

- Solve problems with right triangles and trigonometry

4-10 Applications of Right Triangle Trigonometry

- Right triangle trigonometry

- Draw a triangle and label it
- Solve

4-10 Applications of Right Triangle Trigonometry

- A ladder leaning against a house reaches 24 ft up the side of the house. The ladder makes a 60° angle with the ground. How far is the base of the ladder from the house?

Draw picture

$$\tan 60° = \frac{24}{x}$$

$$\sqrt{3} = \frac{24}{\mathrm{x}}$$

$$x = \frac{24}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3} \approx 13.86\ ft$$

4-11 Bearings and Simple Harmonic Motion

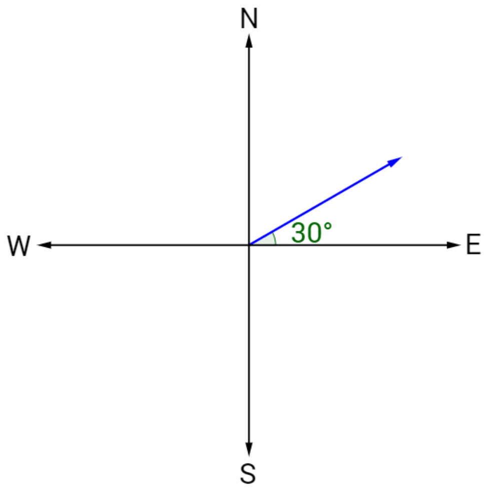
In this section, you will:

- Solve problems involving bearings
- Solve problems involving simple harmonic motion

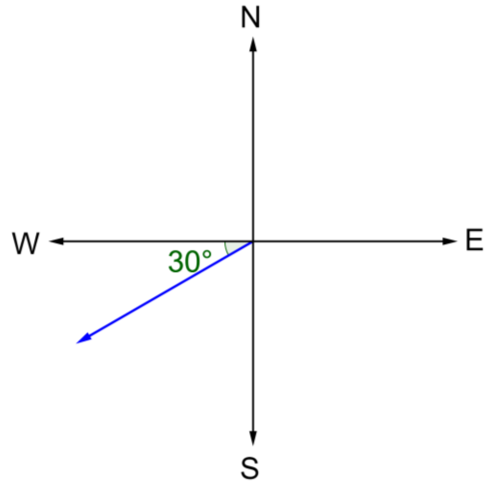
π

4-11 Bearings and Simple Harmonic Motion

- Bearings show direction
- E 30° N



- W 30° S



4-11 Bearings and Simple Harmonic Motion

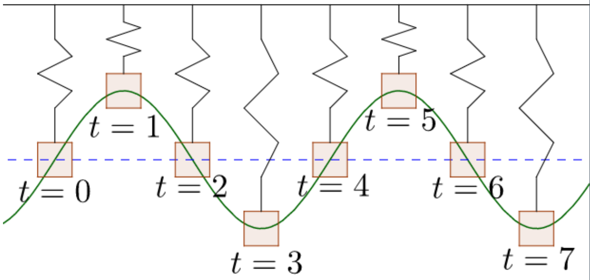
- A sailboat leave a pier and heads due west at 8 knots. After 15 minutes the sailboat tacks, changing course to N 16° W at 10 knots. Find the sailboat's bearing and distance from the pier after 12 minutes on this course.

Draw a diagram and find all components of the N 16° W
Add the x components
Add the y components
Draw a new triangle with those sums
Use Pythagorean theorem to find the hypotenuse
Use inverse tangent to find the angle

3.19 mi at W 37.0° N

4-11 Bearings and Simple Harmonic Motion

- Simple Harmonic Motion (SHM)
- $y = a \sin \omega x$
- $y = a \cos \omega x$



- Period $T = \frac{2\pi}{\omega}$
- Frequency (cycles per second) $f = \frac{\omega}{2\pi}$
- Equilibrium is the centerline

4-11 Bearings and Simple Harmonic Motion

- Find a model for simple harmonic motion with displacement at $t = 0$ is 0, amplitude of 4 cm, and period of 6 sec.

$$a = 4\ cm$$
$$T = \frac{2\pi}{\omega} \rightarrow 6 = \frac{2\pi}{\omega} \rightarrow \omega = \frac{\pi}{3}$$

Starts at 0 so use sine

$$y = a \sin \omega t$$
$$y = 4 \sin\left(\frac{\pi}{3} t\right)$$

4-11 Bearings and Simple Harmonic Motion

- Given the equation for simple harmonic motion

$$d = 4\cos 6\pi t$$

- Find maximum displacement
- Find frequency
- Find value of d when $t = 4$
- Find the least positive value of t for which $d = 0$

4 (amplitude)

$f = \frac{\omega}{2\pi} = \frac{6\pi}{2\pi} = 3$

$d = 4\cos 6\pi 4 = 4$

$0 = 4\cos 6\pi t \rightarrow 0 = \cos 6\pi t \rightarrow \frac{\pi}{2} = 6\pi t \rightarrow \frac{1}{12} = t$